

# Adaptive Anisotropic Regularization of Deformation Fields for Non-Rigid Registration

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**Abstract**—Image registration is a crucial task in many applications and applied in a variety of different areas. In addition to the primary task of image alignment, the deformation field is valuable when studying structural/volumetric changes in the brain. In most applications a regularizing term is added to achieve a smoothly varying deformation field. This can sometimes cause conflicts in situations of local complex deformations. In this paper we present a regularizer, which aims at handling local complex deformations while maintaining an overall smooth deformation field. It is based on an adaptive anisotropic regularizer and its usefulness is demonstrated by two examples, one synthetic and one with real MRI data.

## I. INTRODUCTION

Image registration is a well known problem which aims at finding a deformation field  $\mathbf{d}$  that geometrically transforms one image (source image  $I_S$ ) to fit another image (target image  $I_T$ ), i.e.  $I_T(\mathbf{x}) = I_S(\mathbf{x} + \mathbf{d}(\mathbf{x}))$ .

Image registration algorithms can be divided into two groups, rigid and non-rigid (elastic), based upon their ability to handle different types of deformations. In this paper we focus on non-rigid registration. There exist a number of different non-rigid registration algorithms, which can be further classified according to their theoretical basis (physical models or interpolation and approximation theories). Some well known concepts within non-rigid image registration include thin plate splines [1], viscous fluid flow [2], diffusion models (demons) [3], b-splines and free form deformations [4]. In this paper we will work with a registration and segmentation algorithm known as the Morphon, first presented in [5]. The usability of the Morphon in various situations has been demonstrated in [6], [7].

Image registration is frequently used within medical imaging of the brain. A specific application is when studying structural changes in the brain caused by neurodegenerative diseases or traumas. The obtained deformation field  $\mathbf{d}$  can be used to estimate the local volume change by calculating the Jacobian determinant  $J_D$  of the deformation field [8]. This is known as tensor-based morphometry (TBM).

In order to have reliable results from TBM it is necessary to obtain plausible deformation fields as outputs from the registration process. It is not sufficient just to have a successful registration (defined as visually pleasing and/or a good similarity measure), i.e. a smoother deformation

field is considered more plausible than an irregular, if both achieve a similar registration.

Normally the registration process has some sort of spatially invariant isotropic regularizing term attached to it. The regularizer is constructed to achieve a smoothly varying deformation field, e.g. to prevent tearing or folding of the image (can also be used to control the registration process by describing the characteristics of the material in the images, e.g. its elasticity). However, this approach can cause trouble when local complex deformations are present, which is typical for registration of brain images. The problem is often handled by allowing more degrees of freedom with the result of a visually pleasing registration but a more irregular deformation field. An irregular deformation field can on the other hand be handled by allowing less degrees of freedom with the result of a smoother deformation field but an inferior registration.

The aim of this paper is to present an adaptive anisotropic regularizer which utilizes structural information for handling local complex deformations while maintaining an overall smooth deformation field for the Morphon framework.

## II. METHOD

### A. The Morphon

The Morphon is essentially an algorithm where a source image  $I_S(\mathbf{x})$  is iteratively deformed  $I_D(\mathbf{x}) = I_S(\mathbf{x} + \mathbf{d}(\mathbf{x}))$  until it fits a target image  $I_T(\mathbf{x}) = I_D(\mathbf{x})$ . This process is performed over multiple scales starting on coarse scales to register large global displacements and moving on to finer scales to register smaller local deformations. The algorithm itself consists of the following three steps: *local displacement estimation*, *deformation field accumulation*, *deformation*.

The local displacement estimation is based upon a quadrature phase difference estimation. This is done by filtering the deformed source image and the target image with a set of quadrature filters,  $f_k$  with  $N$  different directions  $\hat{\mathbf{n}}_k$ , where  $I_D(\mathbf{x}) = I_S(\mathbf{x})$  in the first iteration. The filter responses in the  $N$  different directions ( $q_{D_k} = I_D * f_k$ ,  $q_{T_k} = I_S * f_k$ ) describe how edge- or line-like the local neighborhood is coupled with a certainty measure (determined by the phase respectively the magnitude of the filter responses). The outputs are used to compute the local phase difference between the deformed source image

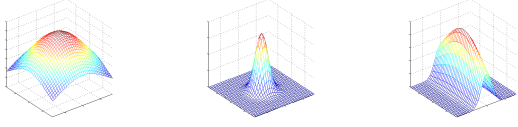


Fig. 1. Left:  $g_{large}$ , Middle:  $g_{small}$  and Right:  $g_{aniso}$ .

and the target image based upon the conjugate products ( $Q_k = q_{D_k} q_{T_k}^*$ ) of the filter responses.

$$d_k = \arg(Q_k) \quad (1)$$

$$c_k = |Q_k|^{1/2} \cos^2\left(\frac{\arg(Q_k)}{2}\right) \quad (2)$$

The phase differences  $d_k$  and the coupled certainties  $c_k$  are then used to compute an incremental displacement field  $\mathbf{d}_i$  and certainty  $c_i$ . This is done by finding the solution to a least square problem defined as:

$$\min_{\mathbf{d}} \sum_{k=1}^N [c_k \mathbf{T}_D (d_k \hat{\mathbf{n}}_k - \mathbf{d})]^2 \quad (3)$$

where  $\mathbf{d}$  is the estimated incremental displacement,  $\mathbf{T}_D = \sum_k |q_{S_k}| \mathbf{M}_k$  is the local structure tensor of  $I_D$  and  $\mathbf{M}_k$  is a projection tensor associated with  $\hat{\mathbf{n}}_k$ . Solving the least square problem is reduced to solving a linear equation system.

$$A\mathbf{d} = \mathbf{b} \quad \Leftrightarrow \quad \mathbf{d} = A^{-1}\mathbf{b} \quad (4)$$

Since this equation system is solved for every point in the image, the elements of  $A$  and  $\mathbf{b}$  are averaged with an isotropic Gaussian kernel in order to improve its robustness.

The incremental displacement field and certainty are then added to the accumulated deformation field  $\mathbf{d}_a$  and certainty  $c_a$  according to:

$$\mathbf{d}_a = \frac{c_a \mathbf{d}_a + c_i (\mathbf{d}_a + \mathbf{d}_i)}{c_a + c_i} \quad (5)$$

$$c_a = \frac{c_a^2 + c_i^2}{c_a + c_i} \quad (6)$$

The last step of an iteration is to use the accumulated deformation field to deform the source image,  $I_D(\mathbf{x}) = I_S(\mathbf{x} + \mathbf{d}_a(\mathbf{x}))$ .

### B. Adaptive anisotropic regularization of the deformation field

Previously it has been suggested to obtain a smoothly varying deformation field by regularizing the accumulated deformation field in each iteration [5]. The regularization is performed by normalized averaging using a Gaussian low pass kernel  $g$  as filter and  $c_a$  as certainty, according to:

$$\mathbf{d}_a = \frac{(c_a \mathbf{d}_a) * g}{c_a * g} \quad (7)$$

A novel approach for handling local complex deformations is to include an adaptive anisotropic regularizing step in the estimation of the incremental displacement

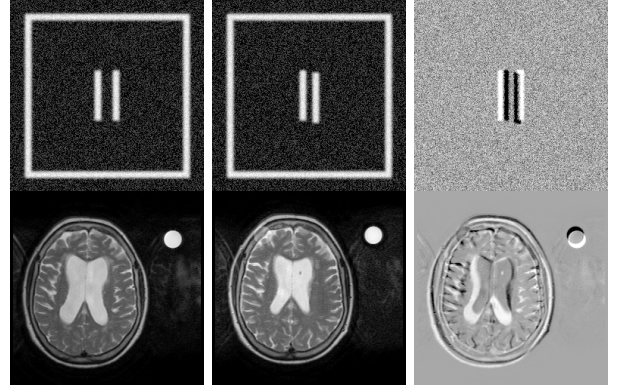


Fig. 2. Top row: Example 1, Bottom row: Example 2, Left:  $I_S$ , Middle:  $I_T$  and Right:  $I_S - I_T$ .

field. The principle behind this suggestion is based upon an idea for adaptive filtering presented in [9].

The idea is to apply an adaptive anisotropic Gaussian kernel instead of a spatially invariant isotropic Gaussian kernel to regularize the elements of the equation system in equation 4. The elements are regularized with a large isotropic kernel  $g_{large}$  if no significant signal is present (case 1). If a single anisotropic structure is present then the elements are regularized with an elliptic kernel  $g_{aniso}$  along the structure (case 2) or they are regularized with a small isotropic kernel  $g_{small}$  if a structure with multiple orientations is present (case 3).

The adaptive regularization is guided by a control tensor  $\mathbf{C}$ . The control tensor  $\mathbf{C}$  is defined as:

$$\mathbf{C} = \sum_{k=1}^N \gamma_k \hat{\mathbf{e}}_k \hat{\mathbf{e}}_k^T \quad (8)$$

$$\gamma_1 = m(\|\mathbf{T}_{Ip}\|, \sigma, \alpha, \beta) \quad (9)$$

$$\gamma_k = \gamma_1 \prod_{l=2}^k \mu\left(\frac{\lambda_l}{\lambda_{l-1}}, \alpha_l, \beta_l\right) \quad k = 2 \dots N \quad (10)$$

where  $\hat{\mathbf{e}}_k$  and  $\lambda_k$  are the eigenvectors and the eigenvalues of  $\mathbf{T}_{Ip} = \mathbf{T}_S * g$ . The purpose of the  $m$ -function is to control the transition between case 1 and 2 whereas the  $\mu$ -function is intended to control the transition between case 2 and 3. The eigenvector  $\mathbf{e}_1$  is used to control the orientation of  $g_{aniso}$ .

The interpretation of the eigenvectors  $\hat{\mathbf{e}}_k$  and the eigenvalues  $\gamma_k$  is straightforward. The eigenvectors are orthogonal and describe the dominant orientations of the local neighborhood. The eigenvalues describe the relationship between the dominant orientations. For the 2D case  $\gamma_1 \approx \gamma_2 \approx 0$  corresponds to case 1,  $\gamma_1 \approx 1$  and  $\gamma_2 \approx 0$  to case 2 and  $\gamma_1 \approx \gamma_2 \approx 1$  to case 3.

$$\gamma_1 \approx 0 \quad \gamma_2 \approx 0 \quad \leftrightarrow \quad \text{Case 1} \quad (11)$$

$$\gamma_1 \approx 1 \quad \gamma_2 \approx 0 \quad \leftrightarrow \quad \text{Case 2} \quad (12)$$

$$\gamma_1 \approx 1 \quad \gamma_2 \approx 1 \quad \leftrightarrow \quad \text{Case 3} \quad (13)$$

Note that  $\|\mathbf{C}\| = 0$  equals case 1 with no significant signal and  $\gamma_1 \approx \gamma_2 \approx 0$ . In the 2D case the different kernels are

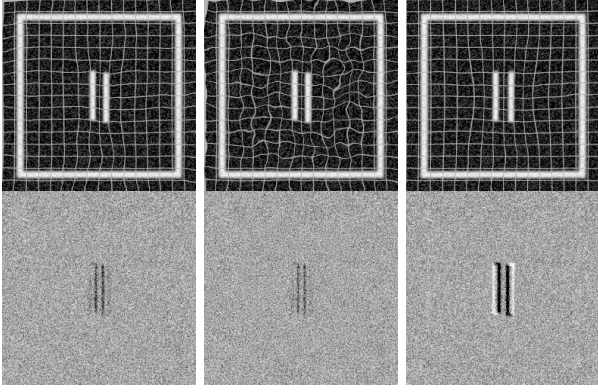


Fig. 3. Top row:  $I_D$ , Bottom row:  $I_D - I_T$ , Left:  $g_{adap}$ , Middle:  $g_{small}$  and Right:  $g_{large}$ .

then added according to:

$$g_{adap} = \|C\| \left( \left(1 - \frac{\gamma_2}{\gamma_1}\right) * g_{aniso} + \frac{\gamma_2}{\gamma_1} * g_{small} \right) + (1 - \|C\|) * g_{large} \quad (14)$$

With the aid of the interpretation of the eigenvectors and the eigenvalues from above it is easy to understand the reasoning behind equation 14. For the case of no predominant orientation  $g_{adap} = g_{large}$ , for the case of one predominant orientation  $g_{adap} = g_{aniso}$  and for the case of several predominant orientations  $g_{adap} = g_{small}$ .

### III. RESULTS

To demonstrate the usefulness of the adaptive anisotropic regularizer two different examples have been tested, one simple synthetic and one using real MRI data. The adaptive regularizer ( $g_{adap}$ ) was compared with using either only  $g_{small}$  or  $g_{large}$  as regularizing kernel. The regularizing filters ( $g_{aniso}$ ,  $g_{small}$  and  $g_{large}$ ) had a kernel size of  $35 \times 35$  in the spatial domain. The two isotropic filters  $g_{small}$  and  $g_{large}$  had a  $\sigma$  of 3 respectively 15. The anisotropic filter  $g_{aniso}$  used  $\sigma = 15$  along the major axis and  $\sigma = 3$  along the minor axis. For example 1 we used 16 anisotropic filters uniformly distributed along a half circle whereas 30 filters were used in example 2.

In example 1 (top row in Fig. 2), two lines have been translated towards each other, one to the right and one down to the left. Both the source and target image have an SNR set to 10dB. Example 2 (bottom row in Fig. 2)

TABLE I  
SIMILARITY MEASURES (NCC, MI) AND AVERAGE GRADIENT DEFORMATION FIELD (G-RMS) FOR EXAMPLE 1 AND 2.

| Example | Regularizer | NCC   | MI    | G-RMS |
|---------|-------------|-------|-------|-------|
| Lines   | $g_{adap}$  | 0.997 | 1.261 | 0.139 |
|         | $g_{small}$ | 0.998 | 1.265 | 0.378 |
|         | $g_{large}$ | 0.984 | 1.185 | 0.127 |
| Brain   | $g_{adap}$  | 0.964 | 1.486 | 0.184 |
|         | $g_{small}$ | 0.967 | 1.518 | 0.305 |
|         | $g_{large}$ | 0.943 | 1.401 | 0.119 |

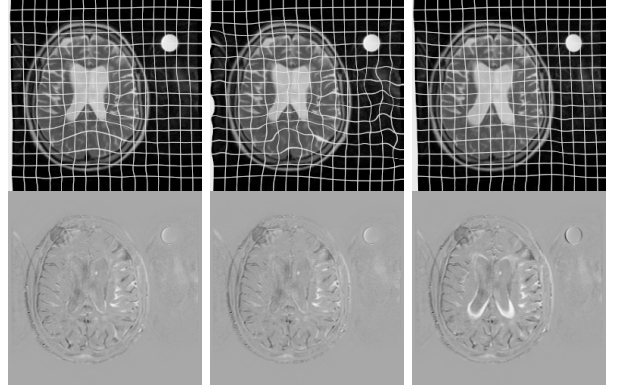


Fig. 4. Top row:  $I_D$ , Bottom row:  $I_D - I_T$ , Left:  $g_{adap}$ , Middle:  $g_{small}$  and Right:  $g_{large}$ .

consists of two MR images<sup>1</sup> displaying a pre- and post-op situation for normal pressure hydrocephalus where a shunt has been inserted to regulate the volume of the cerebral spinal fluid.

The deformed source images  $I_D$  and the difference images  $I_D - I_T$  are displayed in Fig. 3 and in Fig. 4. Fig. 5 and Fig. 6 display the obtained deformation fields  $d(x)$  and the logarithm of the Jacobian determinants,  $\log(J_D)$ . The color-coding of the logarithm of the Jacobian determinant has the following interpretation: green equals expansion, red equals contraction and purple equals folding.

The similarity between the deformed source image and the target image has been measured using normalized cross-correlation (NCC) and mutual information (MI). In the first example this has been estimated without the added noise. The overall smoothness of the deformation field has been measured using the gradient root mean square (G-RMS),  $\sqrt{\frac{1}{N} \sum_{k=1}^N \|\nabla d_k\|}$ , where  $N$  equals the number of pixels in one image.

### IV. DISCUSSION

In both of the examples the  $g_{small}$  kernel gives a result that visually appears to be more correct, see Fig. 3 and Fig. 4. Also the similarity measures suggest the  $g_{small}$  kernel to be favorable, see Table I. However, our objective was to create a smooth deformation field while still handling local complex deformations. The benefits of the adaptive anisotropic regularizer become evident when comparing the deformed grids (Fig. 3 and Fig. 4), the deformation fields and the Jacobian determinant of the deformation fields (Fig. 5 and Fig. 6). The sensitivity to noise for  $g_{small}$  in regions with no visible structures is apparent when observing the deformed grids, especially in Fig. 3 top middle. The smoothness of the deformation field is also confirmed by the G-RMS in Table I.

As predicted earlier a larger regularizing kernel,  $g_{large}$ , is not the remedy to the problems of a small regularizing kernel. The deformation field is much smoother but it fails to handle small local deformations. This can be

<sup>1</sup>These data are work-in-progress acquired by A. Tisell, F. Lundin, O. Dahlqvist Leinhard, G. Leijon, and P. Lundberg.

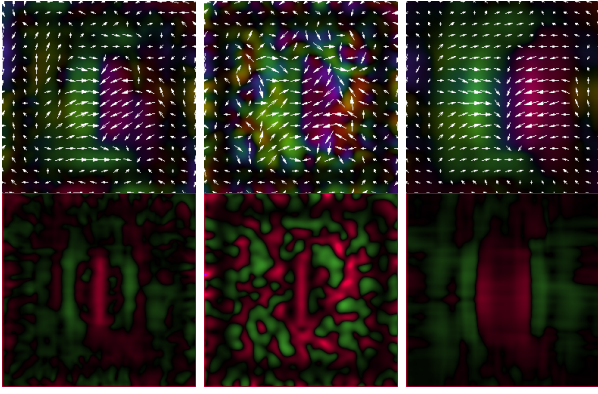


Fig. 5. Top row:  $d(x)$ , Bottom row:  $\log(J_D)$ , Left:  $g_{adap}$ , Middle:  $g_{small}$  and Right:  $g_{large}$ .

seen both in Fig. 3 and Fig. 4 bottom right. In example 1 it fails to bring the two lines together but it also affects the surrounding regions. Similar results can be seen in example 2 where it fails to shrink the ventricles sufficiently.

Another positive aspect of  $g_{adap}$  when comparing it with  $g_{small}$  is evident in the deformation field of example 1 in Fig. 7. Here  $g_{adap}$  produces a displacement for the right line that is almost constant along the whole line (pointing down to the left). However, in the deformation field produced by  $g_{small}$  the points close to the end of the line have a correct displacement (pointing down to the left) but the points in the middle have lost some of their vertical displacement.

Earlier suggestions for dealing with the problem of local complex deformations while maintaining a smooth deformation field can be found in [10], [11]. However, even though both suggestions are based upon an adaptive regularizer controlled by a local tensor they both fall short of utilizing the anisotropy of the tensor.

## V. CONCLUSION

The purpose of the adaptive anisotropic regularizer was to allow local complex transformations while maintaining a smoothly varying deformation field. The success of the regularizer is visible in both examples when comparing with an averaging kernel using only  $g_{small}$  or  $g_{large}$ . Also the result in Table I supports this conclusion since the adaptive regularizer achieves a registration on a similar level as  $g_{small}$  while having a deformation field which is evidently smoother.

Future work includes improving the performance of the adaptive anisotropic regularizer. It also needs to be implemented in 3D. Another important future task is to apply the Morphon to a larger set of medical images containing structural changes and to evaluate the plausibility of the produced deformation fields with the help of medical professionals.

## ACKNOWLEDGEMENT

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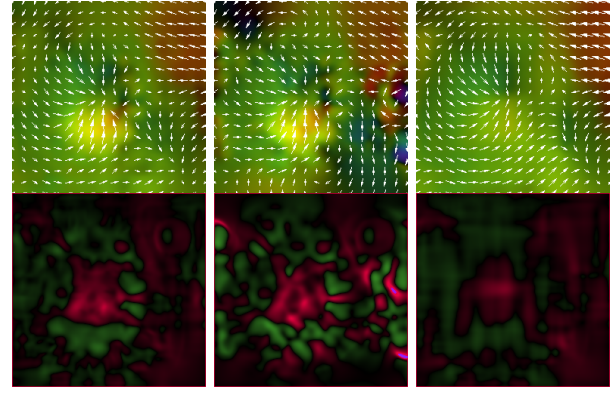


Fig. 6. Top row:  $d(x)$ , Bottom row:  $\log(J_D)$ , Left:  $g_{adap}$ , Middle:  $g_{small}$  and Right:  $g_{large}$ .

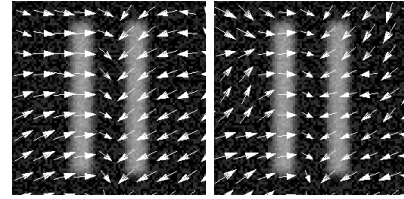


Fig. 7. Close-up of the deformation fields  $d$  from example 1 Left:  $g_{adap}$  and Right:  $g_{small}$ .

## REFERENCES

- [1] A. Goshtasby, "Registration of images with geometric distortions," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 26, no. 1, pp. 60–64, Jan 1988.
- [2] G. Christensen, R. Rabbitt, and M. Miller, "Deformable templates using large deformation kinematics," *Image Processing, IEEE Transactions on*, vol. 5, no. 10, pp. 1435–1447, Oct 1996.
- [3] J.-P. Thirion, "Image matching as a diffusion process: an analogy with Maxwell's demons," *Medical Image Analysis*, vol. 2, no. 3, pp. 243–260, 1998.
- [4] D. Rueckert, L. Sonoda, C. Hayes, D. Hill, M. Leach, and D. Hawkes, "Nonrigid registration using free-form deformations: application to breast MR images," *Medical Imaging, IEEE Transactions on*, vol. 18, no. 8, pp. 712–721, Aug. 1999.
- [5] H. Knutsson and M. Andersson, "Morphons: Segmentation using elastic canvas and paint on priors," in *IEEE International Conference on Image Processing (ICIP'05)*, Genova, Italy, September 2005.
- [6] J. Pettersson, K. L. Palmerius, H. Knutsson, O. Wahlström, B. Tillander, and M. Borga, "Simulation of patient specific cervical hip fracture surgery with a volume haptic interface," *IEEE Transactions on Biomedical Engineering*, vol. 55, no. 4, pp. 1255–65, April 2008.
- [7] A. Parraga, M. D. Craene, J. Pettersson, A. Susin, and B. Macq, "Non-rigid registration methods assessment of 3D CT images for head-neck radiotherapy," in *Proceedings of SPIE Medical Imaging*, San Diego, CA, USA, February 2007.
- [8] M. K. Chung, K. J. Worsley, T. Paus, C. Cherif, D. L. Collins, J. N. Giedd, J. L. Rapoport, and A. C. Evans, "A unified statistical approach to deformation-based morphometry," *Neuroimage*, vol. 14, pp. 595–606, 2001.
- [9] G. H. Granlund and H. Knutsson, *Signal Processing for Computer Vision*. Kluwer Academic Publishers, 1995, ISBN 0-7923-9530-1.
- [10] H. H. Nagel and W. Enkelmann, "An investigation of smoothness constraints for the estimation of displacement vector fields from image sequences," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. PAMI-8, no. 5, pp. 565–593, September 1986.
- [11] R. Stefanescu, X. Pennec, and N. Ayache, "Grid powered nonlinear image registration with locally adaptive regularization," *Medical Image Analysis*, vol. 8, no. 3, pp. 325–342, 2004.