

Computing Generic Rotational Flows Using Warping and its Application to Self-calibration

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Abstract—The computation of smooth optical flows, and their derivatives up to second order, corresponding to the rotation of a generic central camera is addressed. Previous work has shown that it is possible in theory to self-calibrate a generic central camera from only two rotational flow fields, but the quality of the calibration is very sensitive to the accuracy of the estimated flow. In this paper, we show that an improvement in the optical flow computation and the self-calibration results can be obtained by applying image warping techniques to the computation of the optical flow. Experimental results are given both for synthetic and real image data.

I. INTRODUCTION

We define the *optical flow* in a sequence of images as the velocity field in the image domain that transforms one image into the next one [1]. A *generic camera* is a set of pixels and 3D rays in bijective correspondence [2], [3], [4]. Its *calibration* is the assignation map between pixels and rays. We say that the camera is *central* when it has single effective viewpoint; in that case, the calibration map can be defined as a map from the image on the unit sphere [5], [6]. We will consider *smooth generic central cameras*: generic central cameras whose calibration map is a local diffeomorphism (i.e. it is differentiable and its differential map is regular).

The optical flow produced by the infinitesimal rotation of a generic central camera about its centre depends only on the camera calibration and the camera motion; it does not depend on the distance of the scene to the camera. The exact dependence equation was given in [3]. The possibility of exploiting this dependence to self-calibrate a generic central camera using infinitesimal rotations has been studied in several works.

In [5] the authors gave a method to self-calibrate, up to a projective ambiguity, a smooth generic central camera using three independent rotational flows. In [7] the following question was raised: “Are two rotational flows sufficient to calibrate a smooth non-parametric sensor?”; a resolution method was proposed but the question of existence and the ambiguity of the solution remained unanswered.

In [6] a closed-form solution for the self-calibration, up to orthogonal transformation, of a generic central camera from two dense rotational flows was given, answering

affirmatively the previous question. The formulae for the Gram matrix of the two camera angular velocity vectors depended on the flow derivatives up to second order, whereas the formula for the calibration map involves the optical flow derivatives up to first order. In particular, the self-calibration formulae use the second derivatives of the optical flow, and a poor or noisy estimation of the flow can invalidate the self-calibration process; moreover, we need the estimated flow to be smooth in order to correspond to the infinitesimal rotation of a smooth generic central camera.

In most of the literature about the estimation of the optical flow, the main issues are the presence of occlusions and the discontinuities due to different depths on the scene. By contrast, we will assume that the scene and the illumination are static and the camera motion is a pure rotation. Therefore, we do not need to worry about occlusions or discontinuities, but the smoothness of the flow is a critical concern. Being the camera generic, the optical flow cannot be assumed to be affine or to be caused by any parametric camera model.

For these reasons, instead of using already existing algorithms for the estimation of the optical flow, a new method tailored to the problem at hand was proposed in [8]. The optical flow coordinates were parameterized using bicubic splines, so as to guarantee smooth optical flow and derivatives. Besides, the use of splines allowed to model the global/local nature of the generic rotational flow: on the one hand, the flow at a pixel depends on the intensity values in a neighbourhood of the pixel, and, on the other hand, we want optical flows with certain rigidity, defined on a dense subset of the image.

The solution proposed in [8] minimized the squared linearized brightness constancy constraint, penalized with the regularized L_1 norm. This solution was found in a 2-step globally convergent (linear) iterative minimization. The linearity of the brightness constancy constraint was not a good assumption for some of the considered image sequences, with locally big image displacements. To overcome this problem, in the literature, e.g. [9], [10], [11], one often resorts to warping schemes based on iterative Taylor expansions of the image. We apply this scheme in the context of bicubic splines so as to improve optical flow and calibration results in [8].

We will show that it is possible to use splines to determine a warping step that improves the initial estimation of the optical flow based on the linearized brightness constancy constraint. The minimization procedure for the warping computation is similar to that for the flow computation: it consists in iteratively solving a weighted linear system, where the weights are re-computed at each step. By construction, both the optical flow and the warping estimation methods are globally convergent.

II. OPTICAL FLOW CONSTRAINTS

We will review in this section the constraints on the optical flow relating two images of a static scene, acquired at times t and $t+1$ with constant illumination. We denote by $I(p, t)$ the intensity of a pixel p in the image acquired at time t , $w(p) = (v(p), 1)$ the optical flow under study. Except possibly at the image borders, the intensity I remains constant in the direction of the image motion w (*Brightness Constancy Constraint*):

$$I(p + v, t + 1) = I(p, t) . \quad (1)$$

Assuming that the image displacement is small, and that the image intensity is a differentiable function, by linearizing the term at the left of the previous equality, i.e.

$$I(p + v, t + 1) \simeq I(p, t) + \nabla I(p, t)^T \cdot w(p) , \quad (2)$$

we obtain the *Linearized Brightness Constancy Constraint*:

$$\nabla I(p, t)^T \cdot w(p) = 0 . \quad (3)$$

In practice, the equation (3) only constrains the *normal flow*, i.e. the component of the optical flow orthogonal to the image gradient (*aperture problem*), and it is useful only at those pixels where this gradient is not zero. In general, the resolution methods based on (3) either assume the local continuity of the image gradient, or impose the global smoothness of the optical flow and/or its derivatives, or combine both approaches.

As a consequence, a solution $w_0 = (v_0, 1)$ obtained with a method based on the constraint (3) will in general not satisfy (1) or its linearization (2). In this case, we will want to compute Δv_0 , a correction of v_0 such that

$$I(p + v_0 + \Delta v_0, t + 1) = I(p, t) . \quad (4)$$

Since the image $I(p + v_0, t + 1)$ corresponds to the image $I(\cdot, t + 1)$ warped according to the estimated optical flow w_0 , the correction Δv_0 can be seen as the image (not temporal) displacement taking the warped second image to the first image $I(\cdot, t)$.

If we approximate $I(p + v_0 + \Delta v, t + 1)$ by

$$I(p + v_0, t + 1) + \nabla_p I(p + v_0, t + 1)^T \cdot \Delta v_0(p) , \quad (5)$$

using (2) at w_0 , we obtain the following constraint on the correction field Δv_0 :

$$\nabla I(p, t)^T \cdot w_0(p) + \nabla_p I(p + v_0, t + 1)^T \cdot \Delta v_0(p) = 0 . \quad (6)$$

III. PROPOSED METHODS USING BICUBIC SPLINES

Since we are interested in dense optical flows, in order to avoid the aperture problem we need to estimate the optical flow at a pixel while taking into account the local information, rather than the pixel information alone. Using bicubic splines to model the flow we achieve a local/global purpose: on the one hand, all the pixels in a region contribute to the estimation of the same interpolating function; on the other hand, splines can be rigid enough to model dense smooth rotational flows.

Consider a given sequence of images I^i corresponding to the infinitesimal rotation of a generic central camera under the assumptions previously described. We take I_σ^i the convolution of the (discrete) images with a Gaussian filter G_σ of zero mean and deviation equal to σ : $I_\sigma^i = G_\sigma * I^i$. This makes more plausible the assumption of differentiability of the intensity function, used in the deduction of the constraints (3) and (6).

A. Initial Estimation

We call a, b the spline coefficients (column vectors) of the optical flow components: $w = (a^T \cdot s, b^T \cdot s, 1)$, where $s = s(p)$ is a vector of weights that at each pixel p depends solely on the pixel coordinates and the nodes taken to anchor the spline. For each spline, one can choose either a unique bicubic function for the whole image (four nodes at the image vertices) or several ones covering the image, connected with second order contact conditions. In any case, the spline weights s need to be computed only once.

Instead of using a quadratic penalization for the LBCC, which would give a bigger influence to those values with bigger error, we use as penalizer the regularized L_1 norm

$$\psi(x^2) = \sqrt{x^2 + \varepsilon^2} , \quad (7)$$

with ε a small positive constant.

Based on the constraint (3), our initial solution for the optical flow will be the minimum of

$$E_0(a, b) = \sum_{i=1}^{n-1} \int_{\Omega} \psi \left((a^T b^T 1) \cdot M_i(x, y) \cdot \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \right) dx dy , \quad (8)$$

where Ω denotes the image domain and where the tensor matrices $M_i = M_i(x, y)$ can be pre-computed at every pixel and have the form

$$M_i = S \cdot \nabla I_\sigma^i \cdot (\nabla I_\sigma^i)^T \cdot S^T , \quad (9)$$

$$\text{with } S = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

The minimization of the discretization of E_0 can be written as a weighted least squares optimization problem, where the weights (derivatives of the ψ terms) depend on the optical flow. This minimization can be solved by iteratively evaluating its weights and using Cholesky's factorization to linearly compute the spline coefficients. A general proof of the convergence for this iterative strategy can be found in [12].

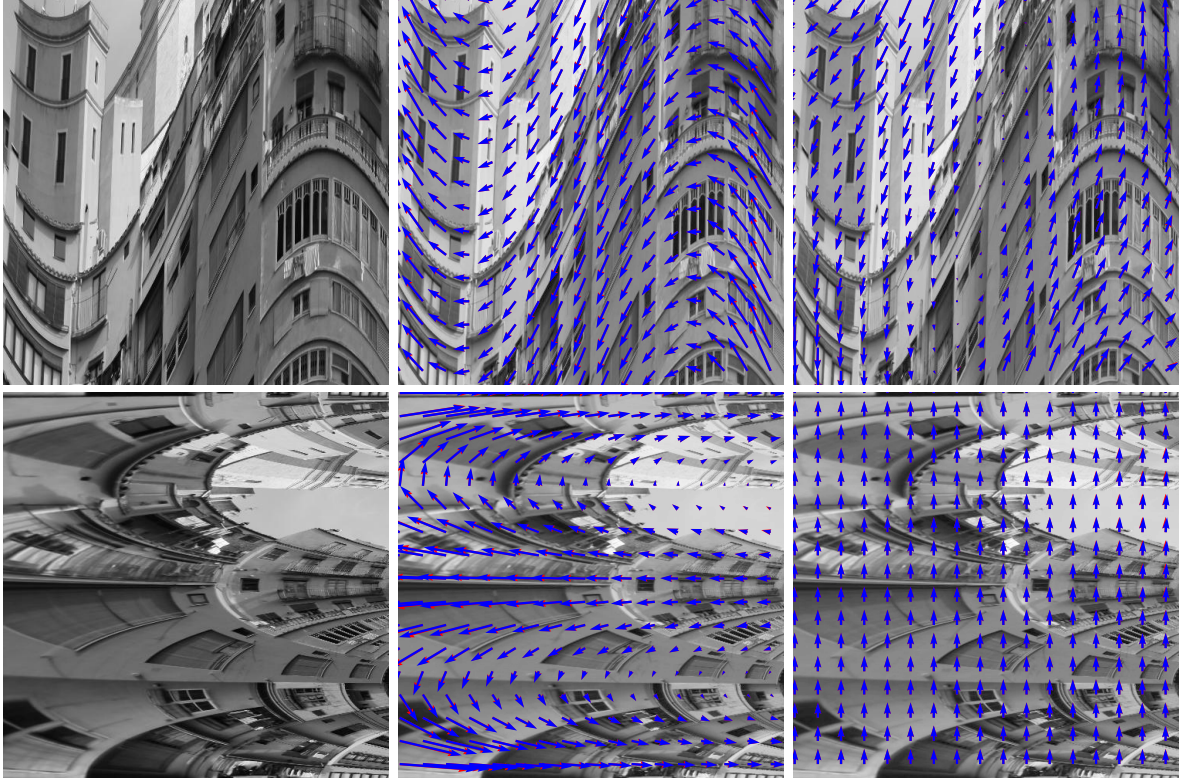


Fig. 1. For the sine (top) and log-polar (bottom) sensors: simulated image (left) and computed flows with Y (centre) and Z (right) rotations.

B. Warping Refinement

Given an initial estimation $w_0 = (v_0, 1)$ for the generic rotational optical flow, we can use splines for the coordinates of the rectification: $\Delta v = (a^T \cdot s, b^T \cdot s)$, where the functions s are the spline weights (as before) and a, b are the spline coefficients to estimate. Based on the constraint (6), our solution for the spline coefficients of the optical flow rectification Δv will be the minimum of

$$\sum_{i=1}^{n-1} \int_{\Omega} \psi \left((\nabla I^i)^T \cdot w_0 + (\nabla_{(x,y)} I^i)^T \cdot S \cdot \begin{pmatrix} a \\ b \end{pmatrix} \right) dx dy, \quad (10)$$

where $\begin{pmatrix} s \\ 0 \end{pmatrix}$ and all the functions are evaluated at (x, y) , excepting $\nabla_{(x,y)} I^i$, which is evaluated at the transformed point $(x, y) + v_0$. The optimization of the discretization of this energy can be also written as a weighted least squares optimization problem; its formulation and resolution is straightforward, once the formulation and resolution of the initial flow estimation have been understood and implemented.

IV. EXPERIMENTAL RESULTS

A. Simulated Images

We simulated 500×500 pixel images acquired by a sensor with focal length of view equal to 250 pixels (i.e. an angular field of view of 90 degrees) composed with parametric distortion models. The chosen distortion

models were defined in $[-1, 1]^2$ as follows [8]:

$$\text{sine}, \quad (u, v) \mapsto \left(u, \frac{1}{2}(v + \sin(\frac{3\pi u}{4})) \right),$$

$$\text{log-polar}, \quad (u, v) \mapsto 10^{\frac{u-1}{2}} (\cos(\pi v), \sin(\pi v)).$$

Respective coordinate changes between $[0, 500]^2$ and $[-1, 1]^2$ were used to define the distortion maps on the images. We used, for each sensor, two simulated sequences of 10 images, corresponding to Y and Z camera rotations with angular velocities

$$\omega_1 = (0, 0.003, 0)^T, \quad \omega_2 = (0, 0, 0.003)^T.$$

1) *Optical Flows*: We computed the optical flows from the simulated image sequences using splines with the proposed two-step optimization method. We fixed the (non-optimized) values $\varepsilon = 10^{-3}$ and $\sigma = 1.5$ for the parameters of the algorithm. For the log-polar Z axis rotation we used a single bicubic function (spline with 4 nodes), whereas for the Y axis rotation of the log-polar sensor we used 3×4 patched bicubic functions. For the rest of flows, we used a spline with 16 nodes (3×3 patched bicubic functions).

In order to compare a computed flow $w_c = (u_c, v_c, t_c)^T$ with its ground truth $w_g = (u_g, v_g, t_g)^T$ we used: the angular error [13], defined by $AE = \arccos \left(\frac{|w_c^T \cdot w_g|}{\|w_c\| \|w_g\|} \right)$, in degrees, and the relative norm error, in %, defined by

$$RNE = \frac{\|w_c\| - \|w_g\|}{\|w_g\|} \times 100.$$

Note that both error measures are very sensitive to noise where the optical flow is close to zero. The average

and standard deviation of the angular error AE and relative norm error RNE of the computed flows are given in Table I. We denote with a 0 subscript the initial estimation, obtained by minimizing E_0 in (8), and with no subscript the final estimation of the flows. We observed that the warping step improves in general the initial flow estimation, taking into account that at those image regions with too big image displacements (for the Y rotations: let/right border regions of the sine sensor, and top/bottom border regions of the log-polar sensor) the assumptions on which our method are based do not hold.

TABLE I
AVERAGE AND STANDARD DEVIATION OF THE AE AND RNE OF THE TWO-STEP ESTIMATIONS FOR THE OPTICAL FLOWS.

Sensor	Axis	μ_{AE}	σ_{AE}	μ_{RNE}	σ_{RNE}
Sine	Y_0	0.230	0.210	3.053	1.440
	Y	0.217	0.230	2.230	1.346
	Z_0	0.466	0.838	1.801	1.231
	Z	0.418	0.760	1.481	1.027
Log-polar	Y_0	0.590	2.003	4.110	4.196
	Y	1.318	2.557	3.903	3.834
	Z_0	0.587	0.986	0.513	0.517
	Z	0.530	0.811	0.334	0.353

2) *Application to Self-calibration*: Following [6], [8], we used the computed pairs of rotational flows to self-calibrate each simulated generic central camera, determining both the camera motion and its calibration. We show in Table II the errors in the estimation of the camera angular velocities; the left errors correspond to the closed-form formulae and the superscript it at the right is used for the errors with the iterative method in [8]; the subscript 0 is used for the initial estimation of the optical flow.

TABLE II
RELATIVE ERRORS (%) CORRESPONDING TO THE NORMS OF THE COMPUTED ANGULAR VELOCITIES AND THEIR ANGLE ω_1, ω_2 .

Errors (%)	$\ \omega_1\ $	$\ \omega_2\ $	$\widehat{\omega_1, \omega_2}$	$\ \omega_1^{it}\ $	$\ \omega_2^{it}\ $	$\widehat{\omega_1^{it}, \omega_2^{it}}$
Sine ₀	4.61	0.40	0.59	3.20	0.60	0.60
Sine	12.69	1.15	0.77	2.67	0.36	1.03
Log-polar ₀	24.70	0.75	5.49	21.93	1.48	1.09
Log-polar	14.82	5.00	0.78	20.71	1.54	0.46

Looking at the left part of the previous table, we observe that the closed-form estimation of the camera angular velocities is sensitive to the errors in the optical flow computation, and that not always the refined flow give better results. In contrast, the results of the iterative estimation of the angular velocities are quite similar, and globally better (smaller norm error of the Gram matrix) for the refined optical flow.

B. Real Images

We used the same rotational sequences and splines as in [8] (see that paper for details), and computed the optical flows from the real images with the algorithm explained here. We show in Fig. 3) an example of rectified image using the self-calibration method of [8].



Fig. 2. Rectified images using the generic self-calibration iterative method with the pairs of computed rotational flows.

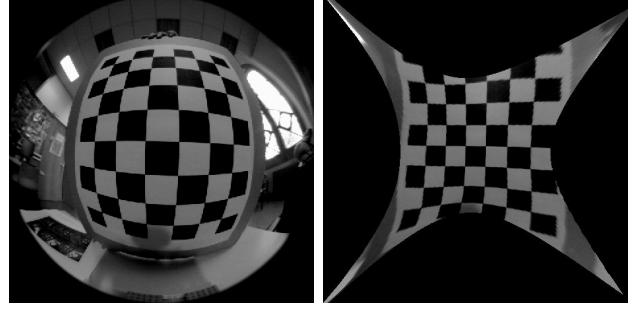


Fig. 3. Fish-eye image of a chessboard pattern covering the area to self-calibrate and rectification of (the interior part of) the image, obtained using the iterative self-calibration algorithm.

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