

The Sum of minimal distances as a useful distance measure for image registration

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Abstract—In this paper we study set distances which are used in image registration related problems. We introduced a new distance as a Sum of minimal distances with added linear weights. Linear weights are added in a way to reduce the impact of single outliers. An evaluation of observed distances with respect to applicability to image object registration is performed. A comparative study of set distances with respect to noise sensitivity as well as with respect to translation and rotation of objects in image is presented. Based on our experiments on synthetic images containing various types of noise, we determine that the proposed weighted sum of minimal distances has a good performances for object registration.

I. INTRODUCTION

To be able to determine the distance between two sets of points is of importance for solving various image analysis related problems, such as image registration, image retrieval and segmentation evaluation. However, to find appropriate distance measure between two sets of points is a difficult task. It is of interest to study and further develop different set distance measures, to be able to select appropriate distances for the different applications.

A metric is a distance on a set X , $d : X \times X \rightarrow \mathbb{R}^+$. Furthermore the metric obeys three properties:

- 1) $d(x, y) = 0 \Leftrightarrow x = y$ (identity of indiscernibles),
- 2) $d(x, y) = d(y, x)$ (symmetry),
- 3) $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality).

For several applications in image processing metricity is not a requirement, and different combinations of conditions (1)–(3) can be well utilized. (see, e.g., [6], [12]).

The Sum of minimal distances, proposed by Eiter and Manila [5], was shown to be a useful distance for object registration problems in our study [10]. We introduce a weighted version of Sum of minimal distances. Different weights are added to the definition of Sum of minimal distances with a purpose to reduce the influence of single outliers.

Image registration is usually considered as a process to find an appropriate geometric transformations between the reference image (model image) and the floating image. In practice, the type of geometric transformations depends on specific application. Having on mind rigid body registration, the appearance of the object has limited degrees of freedom and rigid transformations (translation and rotation) are often sufficient.

We perform a comparative study of the observed distances with respect to translation and rotation of objects in controlled ways. For any real application, it is important that the distance is not too sensitive to noise present in the images. That is the reason why we study behavior of observed distances in the presence of different types of noise added in images.

The present work is organized as follows: Section II gives an overview of related research and connections of the Sum of minimal distances with others set distances. A new set distance as a weighted version of the Sum of minimal distances is presented in section III. Section IV presents an empirical evaluation of monotonicity of each distance with respect to translation and rotation of objects under different noise conditions. Section V concludes the paper.

II. BACKGROUND

There exist a rich literature regarding distance measures used for shape matching, a good survey can be found in [12]. Some of distances measures are in the spatial domain, other rely directly on point-wise comparison of grey levels or colors, while a third class consists of distances which use information both from the spatial and the intensity domains of the images.

Focus of this paper is on the distances between the binary image objects, i.e. distances between crisp sets of points in $\mathbb{Z}^n$, and their usefulness to image registration applications. We restrict our study to the distances introduced in [10], and on the weighted Sum of minimal distances. Also, our study is restricted to distances of linear computational complexity (see [11]), excluding distances of higher complexity such as the Earth Mover's distance or the Netflow distance [12]. All observed distances in this paper can be computed in linear time by using a distance transform.

The most of set distances rely on an underlying distance between the elements of the sets. For the case of image objects, the elements are points in space. A natural choice of distance between points in space is the Euclidean metric, but in general any point distance can be used.

In the following discussion, we assume that the distance between two points a and b is defined as Euclidean distance $d(a, b) = \|a - b\|$.

To simplify notation, we first define a point-to-set distance. Given an underlying distance $d : X \times X \rightarrow \mathbb{R}^+$,

let the distance between $a \in X$ and the non-empty set $B \subset X$ be defined as

$$d(a, B) = \inf_{b \in B} d(a, b). \quad (1)$$

A. Commonly used set distances in image registration

There are many ways how to define the distance between two set of points A and B . In the following we list some commonly used set distances within image processing.

Hausdorff distance: The first proposed set distance and commonly used in image analysis is the Hausdorff metric, d_H , [2]. It is given by

$$d_H(A, B) = \max(\sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A)). \quad (2)$$

On the set of all non-empty compact subsets of X , d_H is a metric. The Hausdorff distance and various modifications of the Hausdorff distance [7],[13], have been used for comparing images.

Metric by Symmetric difference: Another known distance between two sets A, B is based on the symmetric difference of the sets A and B :

$$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

Metric by Symmetric difference is defined as

$$d_{SD}(A, B) = |A \triangle B|, \quad (3)$$

where $|A|$ is the cardinality of the set A . For grey-level or fuzzy images d_{SD} is generalized to a family of Minkowski distances.

Chamfer matching distance: A frequently used distance for object registration problems, introduced in [1] and further developed in [3], is the Chamfer matching distance, d_{CH} ,

$$d_{CH}(A, B) = \sum_{a \in \partial A} d(a, \partial B), \quad (4)$$

as a distance between boundaries of objects. Note that d_{CH} is directed distances, i.e., symmetry is not satisfied.

Sum of minimal distances: The Sum of minimal distances, d_{SMD} , [5],

$$d_{SMD}(A, B) = \frac{1}{2} \left(\sum_{a \in A} d(a, B) + \sum_{b \in B} d(b, A) \right), \quad (5)$$

Triangle inequality is not satisfied and the Sum of minimal distances is not a metric [5].

B. Connections between commonly used set distances and Sum of minimal distances

We note that, as opposed to the other listed distances, the Sum of minimal distances has so far been scarcely used in the field of image processing [4], can be useful in image registration related problems (see [10]). However, it is easy to see that

$$d_{SMD}(\partial A, \partial B) = \frac{1}{2} \left(d_{CH}(A, B) + d_{CH}(B, A) \right),$$

indicating the close relation between d_{SMD} and the more frequently used Chamfer matching distance.

Additionally, if we take the underlying distance d to be the discrete metric, defined on a nonempty set X as

$$d(x, y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}, \quad x, y \in X,$$

then $d_{SMD}(A, B) = \frac{1}{2} d_{SD}(A, B)$.

The Generalized sum of minimal distances is introduced in [10], as

$$d_{SMD}^r(A, B) = \frac{1}{2} \left(\left(\sum_{a \in A} d(a, B)^r \right)^{\frac{1}{r}} + \left(\sum_{b \in B} d(b, A)^r \right)^{\frac{1}{r}} \right), \quad (6)$$

where $r \geq 1$.

More connections between the Generalized sum of minimal distances and commonly used set distances in image processing as Hausdorff metric, Metric by symmetric difference and Chamfer matching distance can be found in [10].

C. Some practical considerations

A desired property of a distance measure for the purpose of object registration is that its value increases with increasing amount of difference between two objects.

Metric by symmetric difference does not take into account information about spatial position of the points. For example, the distance between two non-overlapping objects is the same (as long as their areas do not change), independently of spatial positions or their shapes and its usefulness for object registration problems is limited. Metric based on symmetric difference (3) is easy to compute by counting the number of non-common points. It is also robust with respect to addition or removal of single points.

The Hausdorff distance, which is based on the maximal distance between elements of the set, is very sensitive to one single point, an outlier. Hence, Hausdorff distance is highly sensitive to background noise of binary images and therefore not perfectly suitable as a measure of distance between image objects when point noise may be present. This can also be observed in the evaluation in the section IV.

The Chamfer matching distance as the distance based on the distance between the points on the boundary of objects is sensitive to border noise. Also, the Chamfer matching distance does not differentiate if pixels belong to the object or background.

The Sum of minimal distances is less sensitive to a single outlier than the Hausdorff metric, since it is based on the contributions of all points of the observed sets A and B . However, despite being based on sums over all pixels, one addition pixel still may have a large influence on the Sum of minimal distances. See Figure 1.

III. THE WEIGHTED SUM OF MINIMAL DISTANCES

In addition to the generalization introduced above and existing relations between commonly used set distances, we also observe and discuss the Sum of minimal distances with added linear weights. As we can see in Figure 1, the Sum of minimal distances is not completely suitable distance measure when image contains single outliers.

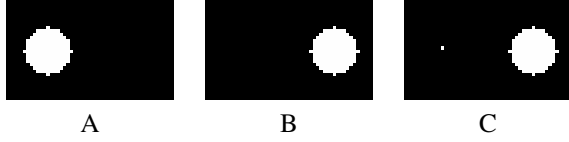


Fig. 1. Despite being based on sums over all pixels, the Sum of minimal distances may still change significantly by the addition of a single point: $d_{SMD}(A, B) = 2878$ and $d_{SMD}(A, C) = 1792$.

In order to reduce influence of a single outliers, we introduce the weighted version of d_{SMD} , such that points which are close to the object have larger weights, while points far away from the object have smaller weights.

Let the number of different distance values $d(a, B) \neq 0, a \in A$ is N_A such that $d_1 < d_2 < \dots < d_{N_A}$. These N_A different values define the same number of classes $A_1, \dots, A_{N_A}$, such that $A_i = \{a \in A : d(a, B) = d_i\}$, $i = 1, \dots, N_A$. We define linear weights as $w_1 = N_A$, $w_2 = N_A - 1, \dots, w_{N_A-1} = 2, w_{N_A} = 1$, in a way that weight w_i is corresponding to the class of points from the set A_i , $i = 1, \dots, N_A$.

In a similar way weights $\delta_j, j = 1, \dots, N_B$ are added to the partition $\{B_1, \dots, B_{N_B}\}$ of the set B .

Then, the Weighted sum of minimal distances is defined as

$$d_w(A, B) = \frac{1}{2} \left(\sum_{1 \leq i \leq N_A} w_i d(a_i, B) + \sum_{1 \leq j \leq N_B} \delta_j d(b_j, A) \right). \quad (7)$$

However, the proposed weighted sum of minimal distances do not solve completely a problem with presence of single outliers in images, but reduce the influence of them. See results in section IV.

IV. EVALUATION

In this section we present the results of an empirical study on the performance of the observed distances. We study how the distance changes as an object is translated or rotated with respect to the original position. To guarantee successful object registration it is desirable that the distance is monotonically non-decreasing with increasing translation and rotation.

Additionally, we perform tests of the observed distances with different noise conditions and study the behavior of the observed distance measures in the presence of noise. To evaluate the noise sensitivity of the distances we add three types of noise to the transformed image.

1. Pepper noise: 5% of pixels inside of the object are randomly assigned to the background pixels.
2. Salt and pepper noise: 1% of the pixels of the whole image change their values.
3. Boundary noise: Pixels value are randomly assigned to inner and outer boundary. The Boundary noise of a given image I is $\varepsilon I \cup (\delta I \cap R)$, where with εI and δI denotes erosion and dilation with a four connected cross, respectively, and R is an image containing random black and white pixels.

Tests are performed on 100 binary images from "A Large Binary Image Database" (LEMS Vision Group,



Fig. 2. Example test images.



Fig. 3. From left: Original noise free image, image with Pepper noise, Salt and pepper noise, and Boundary noise, respectively.

Brown University) [8]. Some of the images can be seen in Figure 2.

We perform the same tests for the newly proposed distance and with the same types of noise as it is done in [10]. Monotonocity of translation, respectively rotation, is fulfilled if distance values are strictly non-decreasing with increasing translation, respectively rotation. Translation is done in a way that each image is translated in a step of one pixel horizontally up to the width of the image. The distance to the non-translated image is computed in each step. Tests for rotation are done such that objects are rotated in positive direction around the centre of gravity of the object. For these tests nearest neighbor interpolation is used for rotation. Range of rotation is from $[0, \frac{\pi}{4}]$. Some of the test images have rotational symmetry and for these objects the occurrence of several minima in the distance plot is unavoidable. Due to this fact, monotonicity on all test objects with respect to rotation can not be reached.

A. Results

For each distance and each type of noise, the results of monotonicity with respect to translation are presented in Table I. The results of translation performed in steps of one pixel horizontally starting from zero and up to the width of the original image width, shown without parenthesis, and 80% of the object width, given in parenthesis.

Similar tests are done for rotation. Results of monotonicity with respect to rotation for all distances and with respect to different noise conditions are shown in Table II. The values that are in parenthesis correspond to the percentage of image fulfilling the monotonicity of distance for rotation of 45 degrees, while the values listed within parenthesis are related for rotations up to 22.5 degrees.

Plots of observed distances for noise free case are shown in Fig. 4. Figure 5 presents how distances change when Salt and pepper noise is added to the image. Magnitude of different distances vary a lot, hence all distances are scaled to the range $[0, 1]$ by dividing them with the respective maximal value.

B. Observations

In [10] it was concluded that distance d_{SMD} provides best performance or very close to the best performance for different noise conditions among the distances analyzed in section 2.

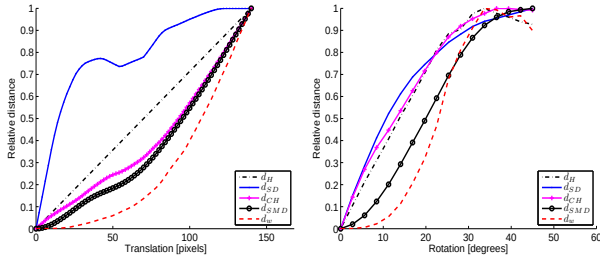


Fig. 4. Plot of distances as a function of translation (left) and rotation (right) of the Airplane image (Fig. 3 top left).

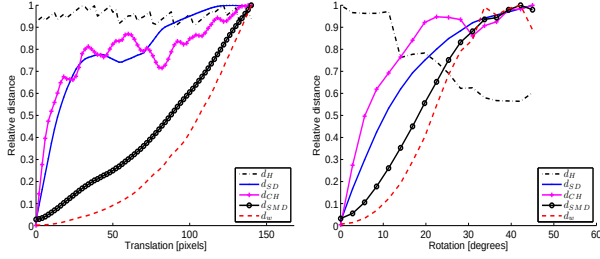


Fig. 5. Plot of distances as a function of translation (left) and rotation (right) of the Airplane image (Fig. 3 top left) with added Salt and pepper noise.

From Tables I-II a number of conclusions can be drawn. Pepper noise only has influence on the d_{SD} , while the other distances are basically unaffected. When Salt and pepper noise is added all over the image, the performance falls significantly. However, performance of d_{SD} and d_{CH} is still far from that of d_{SMD} and d_w . The Boundary noise has a very small impact on the observed distances, even for the Chamfer matching distance, which may be unexpected since this distance is boundary based.

Summarizing results from the tables we conclude that among the existing set distances presented in this paper, the Sum of minimal distances stands out providing the best or very close to best performance for all observed noise cases. Furthermore, the weighted sum of minimal distances, d_w , improve performances of the Sum of minimal distances for all translations and when small rotation is applied.

V. CONCLUSION

We have presented a weighted version of Sum of minimal distances, noted as d_w , as a new set distance useful for object registration which improve the performances of the Sum of minimal distances. Proposed weighted version d_w improves the performances of d_{SMD} for all translations and for smaller rotations.

We intend to perform further studies in this area and explore how to utilize this in an optimal way.

Future work will also include a research of usefulness of the Sum of minimal distances for other image analysis problems, particularly for segmentation evaluation.

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TABLE I
PERCENT OF IMAGES WHERE DISTANCE IS NON-DECREASING W.R.T. TRANSLATION, FOR DIFFERENT TYPES OF NOISE. THE BEST DISTANCE(S) WITH RESPECT TO MONOTONICITY, FOR EACH TYPE OF NOISE, ARE MARKED IN BOLD.

	No noise	Pepper	Salt & pepper	Boundary
d_H	100(100)	100(100)	0(0)	100(100)
d_{SD}	58(61)	59(62)	1(55)	60(63)
d_{CH}	98(98)	98(98)	0(2)	97(97)
d_{SMD}	99(99)	99(99)	97(97)	100(100)
d_w	100(100)	100(100)	97(97)	100(100)

TABLE II
PERCENT OF IMAGES WHERE DISTANCE IS NON-DECREASING W.R.T. ROTATION, FOR DIFFERENT TYPES OF NOISE. THE BEST DISTANCE(S) WITH RESPECT TO MONOTONICITY, FOR EACH TYPE OF NOISE, ARE MARKED IN BOLD.

	No noise	Pepper	Salt & pepper	Boundary
d_H	46(95)	45(95)	0(0)	45(96)
d_{SD}	65(91)	64(92)	62(91)	69(92)
d_{CH}	80(96)	74(95)	1(77)	80(95)
d_{SMD}	79(95)	79(95)	58(94)	78(96)
d_w	63(97)	62(97)	39(96)	62(97)

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REFERENCES

- [1] H.G. Barrow, J.M. Tenenbaum, R.C. Bolles, H.C. Wolf, *Parametric correspondence and chamfer matching: Two new techniques for image matching*, In Proc. 5th Joint Conf. Artificial Intelligence, Cambridge, MA, pp. 659–663, 1977.
- [2] T. Birsan, D. Tiba, *One hundred years since the introduction of the set distance by Dimitrie Pompeiu*, IFIP International Federation for Information Processing, vol. 199/2006, pp. 35–39, 2006.
- [3] G. Borgefors, *Hierarchical Chamfer Matching: A Parametric Edge Matching*, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 10, no. 6, pp. 849–865, 1988.
- [4] D. Deng, *Content-based profiling of image collections: A SOM-based approach*, International Journal of Computers, Systems and Signals, Vol. 5, No. 2, pp. 44–53, 2004.
- [5] T. Eiter, H. Mannila, *Distance measures for point sets and their computation*, Acta informatica, 34, pp. 103–133, 1997.
- [6] A. Hajdu, J. Kormos, B. NJagy, Z. Zörge, *Choosing appropriate distance measurements in digital image segmentation*, Annales Univ. Sci. Budapest Sect. Comp. 24, pp. 193–208, 2004.
- [7] D. P. Huttenlocher, G. A. Klanderman, W. J. Rucklidge, *Comparing Images Using the Hausdorff Distance*, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 15, no. 9, pp. 850–863, 1993.
- [8] B. Kimia, *A Large Binary Image Database*, the LEMS Vision Group at Brown university, 2002.
URL(2009): <http://www.lems.brown.edu/~dmc/>
- [9] R. Klette, A. Rosenfeld, *Digital Geometry*, Morgan Kaufmann, San Francisco, 2004.
- [10] J. Lindblad, V. Čurić, N. Sladoje, *On set distances and their application to image registration*, In Proc. 6th International Symposium on Image and Signal Processing and Analysis (IEEE), pp. 449–454, 2009.
- [11] R. Shonkwiler, *Computing the Hausdorff set distance in linear time for any L_p point distance*, Information Processing Letters, vol. 38, no. 4, pp. 201–207, 1991.
- [12] Veltkamp C. R., *Shape Matching: Similarity Measures and Algorithms*, Shape Modeling International, IEEE Computer Society, 2001.
- [13] C. Zhao, W. Shi, Y. Deng, *A new Hausdorff distance for image matching*, Pattern Recognition Letters Volume 26, Issue 5, pp. 581–586, 2005.