

Robust Detection of Linear Structures Using Non-ring Filters

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Abstract—Many applications in image analysis include the problem of linear structure detection, e.g. segmentation of blood vessels in medical images, roads in satellite images, etc. A simple and efficient solution is to apply linear filters tuned to the structures of interest and extract line and edge positions from the filter output. However, if the filter is not carefully designed, artifacts such as ringing can distort the results and hinder a robust detection. In this paper, we study the ringing effects using a common Gabor filter for linear structure detection, and suggest a method for generating non-ring filters in 2D and 3D. The benefits of the non-ring design are motivated by results on both synthetic and natural images.

I. INTRODUCTION

Linear filters have been successfully applied for line and edge detection in various image analysis applications. For optimal performance, the filters must be designed to fit the structure of interest. This can be conveniently done in the frequency domain, where the center frequency and bandwidth are the most important parameters. Together with desiderata in the spatial domain, a good filter can be found through optimization [1]. However, optimized filters can still suffer from various artifacts, such as *ringing*. In practice, a ringing filter gives repeated indications in a “ring” around the actual structure as illustrated in Fig. 2(d). This can create problems not only for proper detection of single lines and edges, but the ringing of neighboring lines could severely interfere with the filter responses of the actual structures.

One-dimensional non-ring filters have previously been proposed in [2]. The main contribution of this paper is the extension to higher dimensions. Our approach is based on a 1D design which is “rotated” in the higher dimension. To keep illustrations intuitive, we give most examples using 2D filters and images, but show a proof-of-concept case in 3D.

Sec. II will start by introducing filters for linear structure detection. Using a common Gabor filter we will give intuitive examples to why the ringing can be troublesome for segmentation applications. Next, Sec. III will describe non-ring 1D designs, which will be used for higher dimensions. Experiments on natural images will be presented and discussed in Sec. IV and Sec. V, followed by conclusions and ideas for future work.

II. FILTERS FOR LINEAR STRUCTURE DETECTION

There exists many different filters which can be applied for line and edge detection ranging from simple Laplacian operators to Gabor filters [3], to banks of oriented filters such as steerable filters [4] or other types of quadrature filter pairs [2], [5]. For the examples in this paper we will use a Gabor filter, specified in the 1D frequency domain by:

$$G_1(u) = e^{-(u-u_0)^2/2\sigma^2} \quad (1)$$

where u_0 is the center frequency and σ is the variance corresponding to the bandwidth. A filter with $u_0 = 2\pi/7$ and $\sigma = 0.22$ is depicted in Fig. 1. Note that this filter is represented by an even-odd filter pair in the spatial domain, and that the phase wraps around several times during the filter transition. A Gabor filter in 2D is given by [6]:

$$G_2(\mathbf{u}) = e^{-||\mathbf{u}-\mathbf{u}_0||^2/2\sigma^2} \quad (2)$$

where u_0 are center frequencies along the coordinate axes. A 2D Gabor filter, and any other type of quadrature pair, can be used for linear structure detection in images. The relation between the even (real) and odd (imaginary) filter responses indicates whether the structure is “line-like” or “edge-like”. The center of a line can be found at positions with a maximum real part, whereas edges are found at imaginary maxima. Equivalently, the center of a line can also be detected by the *zero-crossing* of the imaginary part, while edges are located at the zero-crossing of the real part. The latter formulation is more useful algorithmically, since the detection of zero-crossings can be more robust than maxima or minima searches. In particular, in a multi-scale approach such as [7], zero-crossings of the real part are important features.

A. The ringing effect

For illustration of filter based edge detection, we use a synthetic image of a disk displayed in Fig. 2(a). To cover the orientation space, we need to filter the image using at least 3 differently oriented filters in 2D or 6 filters in 3D. Since there is no need to discriminate between the two different edge events of either stepping *into* or *out of* an object, we rectify the odd filter by taking the absolute value of the imaginary part of the filter response. This makes it possible to simply sum the responses from

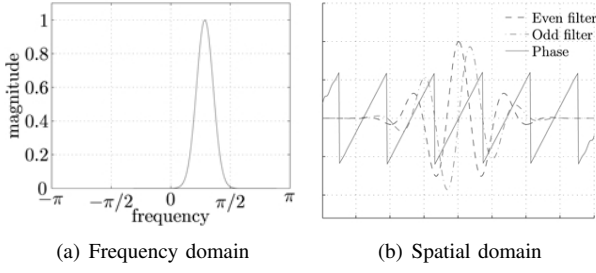


Fig. 1. 1D Gabor filter

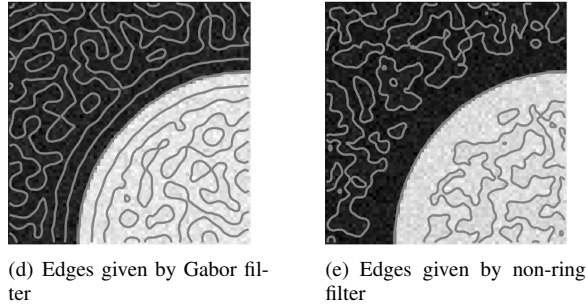
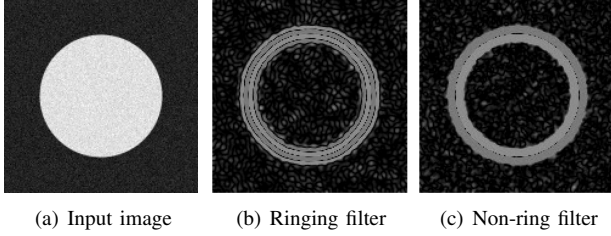


Fig. 2. Effects of ringing on disk

different orientations as described in [7]. We can then identify the edges of an object by locating the zero-crossings of the real part of the filter response. Performing the filtering on the disk in Fig. 2(a) using a Gabor filter gives the result in Fig. 2(b), where green and red colors indicate positive and negative values respectively. The detected edges (zero-crossings of the result) are shown in Fig. 2(d), overlaid the original image. A number of repeated edge structures can be observed around the disk, and be detected as false positives. This shows the importance of proper filter design when applying the presented edge detection strategy. Even more complex ringing patterns can be expected for natural images, where the ringing of strong structures could annihilate the actual response of neighboring weak structures. Also the point in Fig. 3(a), which has no structure by definition, may cause structured ringing from a ringing filter as seen in Fig. 3(b). This may cause unstructured noise to become structured after the filtering, and mistaken for relevant objects in the processing steps.

III. NON-RING FILTERS

The design of 1D non-ring filters was first proposed in [2]. A general formulation is given in the spatial domain by:

$$f(\xi) = g'(\xi)e^{i\pi g(\xi)/g(R)} \quad (3)$$

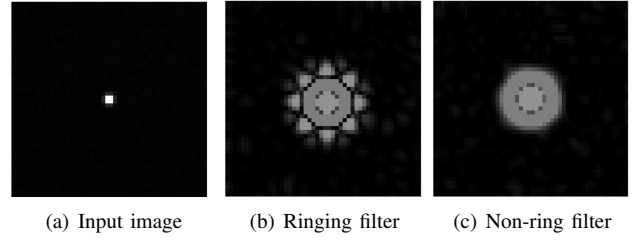


Fig. 3. Effects of ringing on point

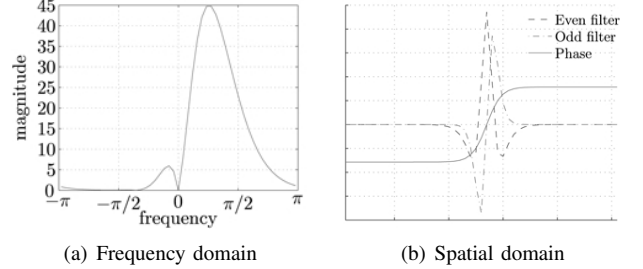


Fig. 4. 1D non-ring filter

where $g(\xi)$ is the *phase function* and R is the radii of the spatial support of the filter. The function $g(\xi)$ can be any monotonous antisymmetric function. We note that $g'(\xi)$ controls the envelope of the filter. A simple choice is $g(\xi) = \xi$, $\xi \in [-\pi, \pi]$ which leads to a box-shaped envelope, i.e. $g'(\xi) = 1$, $\xi \in [-\pi, \pi]$. However, a smooth filter envelope is usually desired, so in this paper we use the hyperbolic tangent function such that $g(\xi) = \tanh(\xi)$. We also add a scaling parameter α , to control the frequency contents of the filter:

$$f(\xi) = \alpha\pi (1 - \tanh(\alpha\xi)^2) e^{i\pi \tanh(\alpha\xi)} \quad (4)$$

where $\xi \in [-\pi, \pi]$ and $\alpha > 0$. A plot of this filter using $\alpha = 2$ is shown in Fig. 4. Note that the phase is monotonous and does not wrap around, which is a characteristic of a non-ringing filter (cf. the ringing filter in Fig. 1).

A. Non-ring filters in higher dimensions

To construct higher dimensional non-ring filters, we assume the filter to be spherically separable into a radial function $R(\rho)$ and a directional function $D(\hat{\mathbf{u}})$ [5], where $\rho = \|\mathbf{u}\|$ and $\hat{\mathbf{u}} = \mathbf{u}/\|\mathbf{u}\|$ is the normalized frequency coordinate. We use a directional function suggested in [5] for a filter direction $\hat{\mathbf{n}}_k$:

$$D_k(\hat{\mathbf{u}}) = (\hat{\mathbf{u}} \cdot \hat{\mathbf{n}}_k)^2 \quad (5)$$

It can be noted that this function varies as $\cos^2(\phi)$ where ϕ is the angle between $\hat{\mathbf{u}}$ and $\hat{\mathbf{n}}_k$. While $D(\hat{\mathbf{u}})$ determines the directional restriction, $R(\rho)$ defines the radial shape of the filter in the frequency domain. In higher dimensions we need a correspondence to the negative frequencies $u < 0$ in 1D. We define this using the filter direction, allowing a negative parameter ρ to the radial function:

$$\rho = \begin{cases} \|\mathbf{u}\| & \text{if } \hat{\mathbf{u}} \cdot \hat{\mathbf{n}}_k > 0 \\ -\|\mathbf{u}\| & \text{otherwise} \end{cases} \quad (6)$$

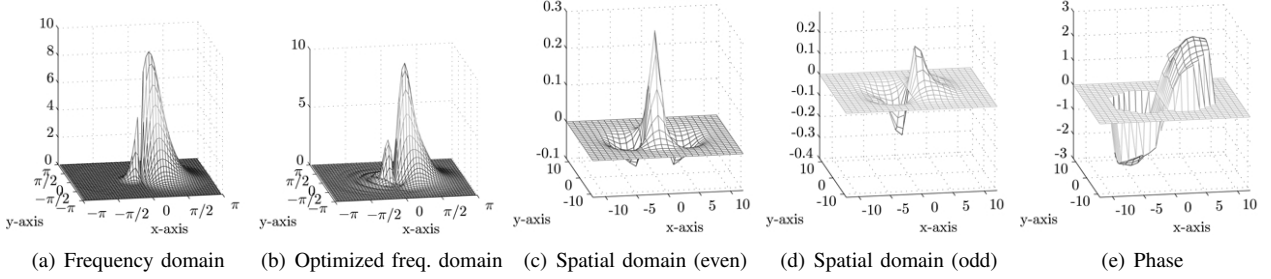


Fig. 5. 2D non-ring filter

To compute $R(\rho)$, we use the Fourier transform of the 1D filter in Eq. (4), $\mathcal{F}[f(\xi)] = F_1(u)$ (see Fig. 4(a)):

$$R(\rho) = \frac{F_1(\rho)}{|\rho|^{n-1}} \quad (7)$$

where n is the number of dimensions. The division by $|\rho|^{n-1}$ is needed to compensate for the fact that when the 1D filter is used as a radial function for filters in higher dimensions, the frequency response will grow proportionally to $|\rho|^{n-1}$. This construction can be interpreted as a “rotation” of the 1D Fourier transform in a higher dimension. Finally the filter is created from the combination of $R(\rho)$ and $D(\hat{u})$:

$$F_n(\mathbf{u}) = R(\rho)D(\hat{u}) \quad (8)$$

A 2D non-ring filter generated using this procedure is shown in Fig. 5(a). To convert this into the spatial domain given an arbitrary kernel size, we use the optimization procedure presented in [1]. The optimized result in Fig. 5(b), which closely resembles the input function in Fig. 5(a), gives the spatial filters in Fig. 5(c,d). As can be verified in Fig. 5(e), this is a non-ring filter with no wrap around of the phase. The examples in Fig. 2(e) and Fig. 3(c) also verify this result in practice.

IV. EXPERIMENTS

For the experiments, we apply the previously described 2D filters on the classic “camera man” image in Fig. 6(a) and a retinal image from the DRIVE database [8] in Fig. 7(a). We use the parameter $\alpha = 3$ for the non-ring filter, while the Gabor is designed such that the center frequency matches the mean frequency of the non-ring filter, resulting in $u_0 = 2\pi/7$. For our simple Gabor design, it is not possible to match the bandwidth of the two filters, so we pick $\sigma = 0.22$ which gives the largest bandwidth without significant DC response. The real part of the filter responses are shown in Fig. 6(b,c) and Fig. 7(b,c). Fig. 6(d,e) and Fig. 7(d,e) show the edges (zero-crossings of the real part) detected by the corresponding filter. To show the generalization to 3D, we generate corresponding 3D Gabor and non-ring filters which are applied to a synthetic 3D ball. The edge surfaces (zero-crossings of the real part) for half the ball are displayed in Fig. 8. As a complete case, the Gabor filter and the non-ring filter have also been applied on the generic Shepp-Logan head phantom (described in [9] and

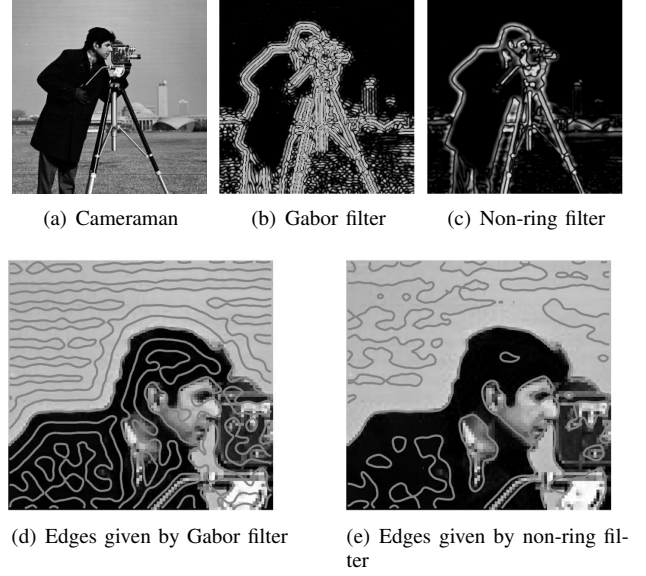


Fig. 6. Filtering of cameraman image

available in Matlab® in the Image Processing Toolbox. As for the 2D pictures of the cameraman and of the retinal image, a slice of the 3D volume from the head phantom along with the edges from both the Gabor and the non-ring filter are displayed in 2D in Fig. 9(a,b,c). Notice that Fig. 9(b) and Fig. 9(c) are zoomed versions of the Gabor and non ring filter responses on the middle slice shown Fig. 9(a). A cropped volume representation of the real part of the filter response for both the Gabor and the non ring filter is also represented in Fig. 9(d,e), followed by their surface representation (shown as half volume to better visualise the interior of the head phantom).

V. RESULTS

From the synthetic examples in Fig. 2, Fig. 3, Fig. 8 and Fig. 9, it is shown how the ringing of a poorly designed filter can create multiple detection patterns in a “ring” around the actual structure. This effect is visible in natural images as shown in Fig. 6(d) and Fig. 7(d), where we see a number of edge indications parallel to the actual edges. Also, as was noted in Fig. 3(b), the ringing of an unstructured object such as a point can create a false indication of structure. In a noisy image, this effect can create structured noise in the filter response which can be difficult to separate from “actual” structure.

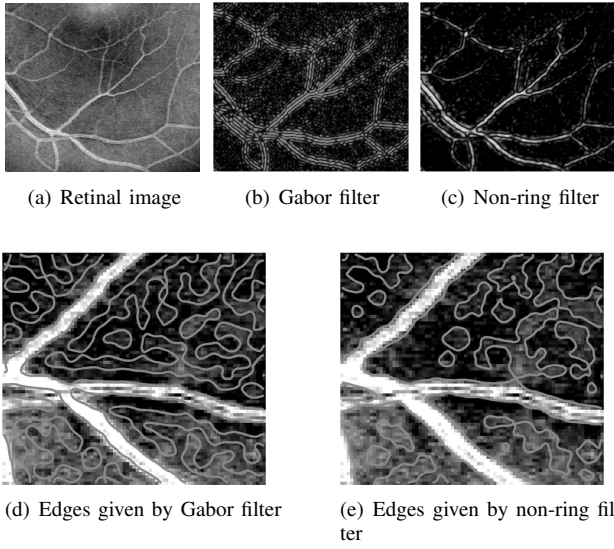


Fig. 7. Filtering of retinal image

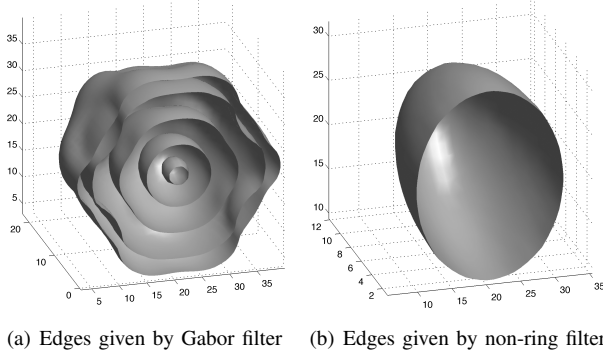


Fig. 8. 3D filtering of ball

Fig. 9(f) is a clear representation of such an effect where it is rather challenging to make a distinction between the difference objects present in the head phantom dataset. On the opposite, it is very clear from the results on the non-ring filter response represented in Fig. 9(g) where the shapes can easily be recognised when comparing to the original 2D slice displayed in Fig. 9(a).

VI. CONCLUSIONS AND FUTURE WORK

We have presented a simple and general way of designing multidimensional filters which do not suffer from ringing artifacts. The design is based on a 1D filter with the requirement that the filter envelope is the derivative of the phase function. This filter is then used as a radial function in higher dimensions and modulated with a directional function. We compare with a Gabor filter, which generates apparent ringing artifacts.

Future work include the parametrization of a non-ring filter which allows for direct specification of bandwidth and center frequency. We will also further investigate the behavior of these filters in 3D.

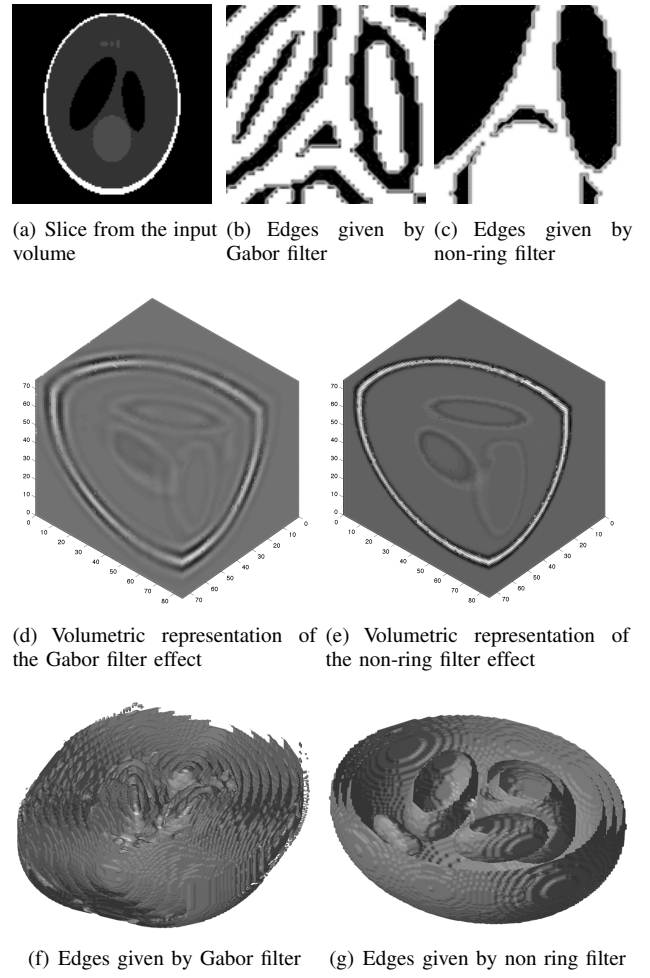


Fig. 9. Filtering of generic Shepp-Logan head phantom

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